GCSE Maths – Ratio, Proportion and Rates of Change

General Iterative Processes (Higher Only)

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out questions relating to general iterative processes. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A – Higher Only

Worked Example

At the start of day 1, James has 50 tyres in his tyre shop. Each day 40% of his tyres are sold. He receives a delivery of 15 new tyres at the end of each day. How many tyres will he have at the end of day 6?

Step 1: Identify the variables.

$$T = number\ of\ tyres$$

$$T_0 = initial\ number\ of\ tyres$$

$$T_n = \ number\ of\ tyres\ at\ the\ end\ of\ the\ day$$

Step 2: Set up a formula.

The formula is specific to each question and requires manipulation of the context provided in the question. Not all questions will be similar to this – but every question involving iteration can be put into its own formula!

In this case, each day the number of tyres decrease by 40%.
This is equivalent to $1 - 0.4 = 0.6$ *of the day's starting value. We also know that an extra 15 tyres are delivered each day. This can be written as an addition in the formula.*

$$\boldsymbol{T_{n+1} = 0.6(T_n) + \ 15}$$

Step 3: Use iterations to find number of tyres at the end of day 6

$T_1 = 0.6(50) + \ 15 = 45$

There are 45 bouquets at the end of day 1.

$T_2 = 0.6(45) + \ 15 = 42$

$T_3 = 0.6(42) + \ 15 = 40.2 = 40\ (nearest\ full\ tyre)$

$T_4 = 0.6(40) + \ 15 = 39$

$T_5 = 0.6(39) + \ 15 = 38.4 = 38\ (nearest\ full\ tyre)$

$T_6 = 0.6(38) + \ 15 = 37.8 = 37\ (nearest\ full\ tyre)$

At the end of day 6, James will have 37 tyres in his shop.

Guided Example

Frank owns a restaurant in Manchester. He plans to refurbish his restaurant which would cost around £1000. He plans to start saving all his monthly profit to cover the refurbishment cost. Starting from January 2021, his monthly profit was enough to cover 5% of the total refurbishment cost. Assuming his monthly profit was doubled after every month, by which month of the year would he be able to refurbish his restaurant?

Step 1: Identify the variables.

P = Frank's profit

P_0 = Frank's profit in January

P_n = Frank's profit at the end of each month starting February

Step 2: Set up a formula.

In the first month of January, the profit is 5% of the refurbishment cost. After each month, the monthly profit is doubled. Hence the formula is :

$$P_{n+1} = 2(P_n)$$

This formula should only be used to calculate the profit starting February since in January, there are no doubling of profit

$[5\% = 5/100 = 0.05]$

$P_0 = 0.05 \times 1000 = 50$

calculate the profit in January first

Step 3: Use iterations to find the number of month he needs to cover the refurbishment cost.

$n=0$, February $\rightarrow$ $P_1 = 2(50) = 100$

$n=1$, March $\rightarrow$ $P_2 = 2(100) = 200$

$n=2$, April $\rightarrow$ $P_3 = 2(200) = 400$

$n=3$, May $\rightarrow$ $P_4 = 2(400) = 800$

stop calculation and add all the previous amounts to see if the total is more than £1000

Total profit made so far including January :

$= £50 + £100 + £200 + £400 + £800$

$= £1550$

total is more than £1000

Hence, by May 2021, Frank would be able to refurbish his restaurant.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Tom invested £200 to an international bank account. The bank guarantees a 9% bonus at the end of each year. However, 1.5% bank fees will be deducted from the remaining balance after the bonus has been added. How many years does Tom need to wait to have more than £300 in his bank account?

M = total money in Tom's bank account

M_0 = initial money Tom invested

M_n = total money in Tom's bank account at the end of the year

$(100\% + 9\% = 109\% \rightarrow 109 \div 100 = 1.09)$

$\boxed{100\% - 1.5\% = 98.5\% \quad 98.5\% \div 100 = 0.985}$ multiplier

Formula $\rightarrow$ $M_{n+1} = (1.09 \times M_n) \times (0.985)$

$M_1 = (1.09 \times 200) \times 0.985 = 214.73$

$M_2 = (1.09 \times 214.73) \times 0.985 = 230.54$

$M_3 = (1.09 \times 230.54) \times 0.985 = 247.52$

$M_4 = (1.09 \times 247.52) \times 0.985 = 265.75$

$M_5 = (1.09 \times 265.75) \times 0.985 = 285.33$

$M_6 = (1.09 \times 285.33) \times 0.985 = 306.34$ $\rightarrow$ stop calculation as amount exceeds £300

Tom needs to wait for 6 years to have more than £300 in his bank account

2. Kevin owns a fruit stall. At the start of day 1, he has 200 oranges for sale. Each day, 20% of his oranges are sold. On day 7, no sales are made but he donates 15% of his remaining oranges to the homeless and receives an additional delivery of 20 new oranges. What is the total number of oranges that Kevin has at the end of day 7?

F = number of oranges left

F_0 = initial number of oranges Kevin has in his stall

F_n = number of oranges left at the end of the day

Only use Formula from Day 1 to 6

Formula : $F_{n+1} = 0.8(F_n)$

$F_1 = 0.8(200) = 160$

$F_2 = 0.8(160) = 128$

$F_3 = 0.8(128) = 102.4 \approx 102$

$F_4 = 0.8(102) = 81.6 \approx 82$

$F_5 = 0.8(82) = 65.6 \approx 66$

$F_6 = 0.8(66) = 52.8 \approx 53$

round to whole numbers as oranges can only exist as whole numbers

no sales are made but 15% are donated + 20 new delivery

Day 7 $= (53 \times 0.85) + 20 = 65.05 \approx 65$ oranges

Kevin has 65 oranges left at the end of Day 7

3. Sarah bought a used car worth £500. Every year, the price of the car decreases by 8%. What would be the price of the car after 10 years? Starting from which year will the price of the car drop below £300?

P = price of used car
P_0 = initial price of used car
P_n = price of used car at the end of the year

$P_{n+1} = 0.92 \times P_n$

$P_1 = 0.92 \times 500 = 460$
$P_2 = 0.92 \times 460 = 423.2$
$P_3 = 0.92 \times 423.2 = 389.34$
$P_4 = 0.92 \times 389.34 = 358.20$
$P_5 = 0.92 \times 358.20 = 329.54$
$P_6 = 0.92 \times 329.54 = 303.18$
$P_7 = 0.92 \times 303.18 = 278.92$
$P_8 = 0.92 \times 278.92 = 256.61$
$P_9 = 0.92 \times 256.61 = 236.08$
$P_{10} = 0.92 \times 236.08 = 217.19$

The price of the car after 10 years is £ 217.19. The price of the car drops below £ 300 starting from the 7th year.

4. Amy needs to knit 150 socks in 15 days. At the start of day 1, she was able to knit 20 socks. However, her progress decreases by 10% each day. The minimum number of socks she will get done is 5 each day. Is she able to finish knitting 150 socks in 15 days?

S = Number of socks Amy has
S_0 = Number of socks at the end of day 1
S_n = Number of socks at the end of the day (starting Day 2)

$S_{n+1} = 0.9(S_n)$

$S_1 = 0.9(20) = 18$
$S_2 = 0.9(18) = 16.2 = 16$ round to whole number
$S_3 = 0.9(16) = 14.4 = 14$
$S_4 = 0.9(14) = 12.6 = 13$
$S_5 = 0.9(13) = 11.7 = 12$
$S_6 = 0.9(12) = 10.8 = 11$
$S_7 = 0.9(11) = 9.9 = 10$
$S_8 = 0.9(10) = 9$
$S_9 = 0.9(9) = 8.1 = 8$
$S_{10} = 0.9(8) = 7.2 = 7$
$S_{11} = 0.9(7) = 6.3 = 6$
$S_{12} = 0.9(6) = 5.4 = 5$
$S_{13} = 0.9(5) = 4.5 = 5$
$S_{14} = 0.9(5) = 4.5 = 5$ stop calculation to calculate the number of socks she has knitted so far

Socks Amy has from Day 1 to Day 15:
$= 20 + 18 + 16 + 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 5 + 5$
$= 159$ socks

more than target : 150

Amy is able to finish knitting 150 socks within 15 days